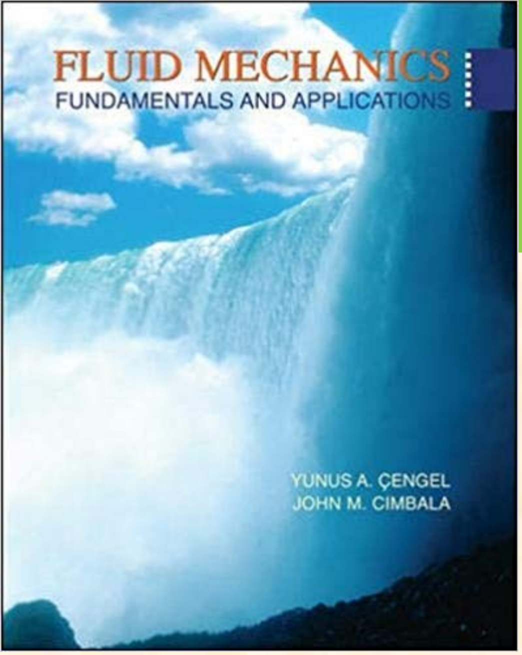

YUNUS A. ÇENGEL and JOHN M. CIMBALA, "Fluid Mechanics: Fundamentals and Applications",1st ed., McGraw-Hill,2006.

Course name

Incompressible Fluid Mechanics

Lecture-01- Chapter-03

Fluid flow concept and Basic equations

Lecture slides by

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University of Tikrit

Outline

- ***Definitions***
- ***Control volume and system representation***
- ***Reynolds transport theorem (RTT)***
- ***Continuity equation***
- ***Energy equation***
- ***Bernoulli equation***
- ***Examples***
- ***Homeworks:***

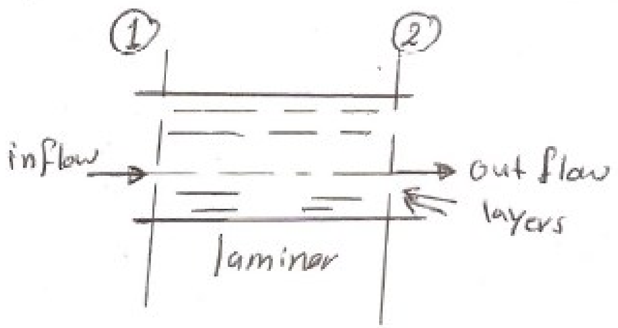
3.1 Definitions

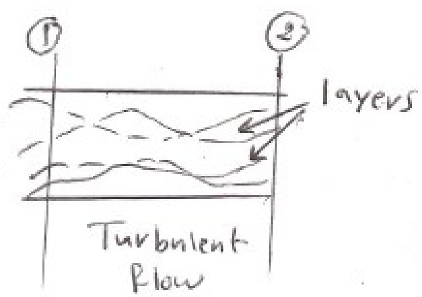
1- Laminar flow

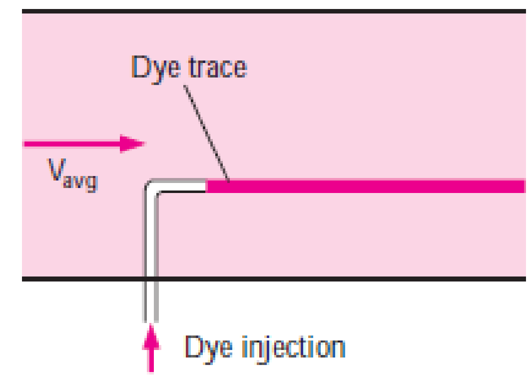
Fluid particles moves along smooth paths in laminar or layers

2- Turbulent flow

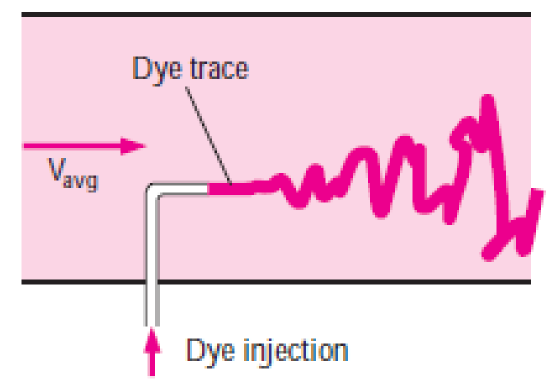
Fluid particles moves in very irregular paths, causing change in momentum from portion of fluid to another







(a) Laminar flow



(b) Turbulent flow

$$\text{Reynolds number} = \frac{\rho D V}{\mu} = \frac{D V}{\nu}$$

ν ← Kinematic viscosity

$0 < Re < 2000$ laminar

$2000 < Re < 4000$ Transition

$Re > 4000$ Turbulent

Hint:- $\nu = \frac{\mu}{\rho}$

3.1 Definitions

②

3- uniform flow

occurs, when the velocity Vector of the fluid at every point is identically the same for any given instant:-

$$\frac{\partial V}{\partial s} = 0 \;,\; \frac{\partial V}{\partial t} = 0 \;, \ldots$$

4- non-uniform flow

occurs when the velocity vector changes for any given instant

$$\frac{\partial V}{\partial s} \neq 0 \;,\; \frac{\partial V}{\partial t} \neq 0 \;, \ldots$$

5- Steady flow

occurs when conditions of the fluid at a Point don't change with time.

$$\frac{\partial V}{\partial t} = 0 \;,\; \frac{\partial P}{\partial t} = 0 \;,\; \frac{\partial \rho}{\partial t} = 0 \;, \ldots$$

6- Unsteady flow

occurs when condition of the fluid at a Point change with time

$$\frac{\partial}{\partial t} \neq 0$$

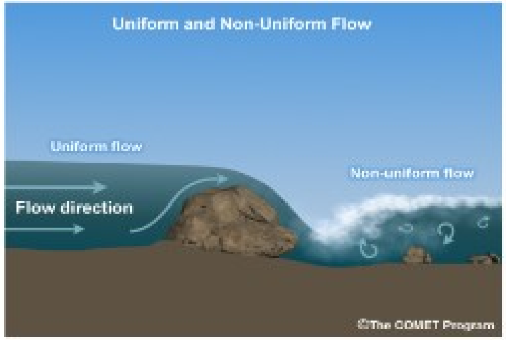
7- Average velocity

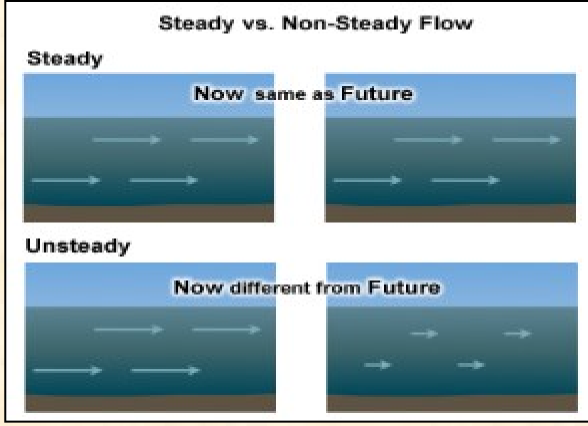
$$V = \frac{Q}{A} \;,\; V = \frac{ds}{dt}$$

Q:- flow rate m^3/s

A:- Cross-Section area

Uniform/Non Uniform flow





https://edurev.in/studytube/Chapter--Flow-Measurement--PPT--Semester--Engineer/867fbec9-eefb-4a7d-9163-f6d25b176b3c_t

3.2 Control volume and system representation

System

Collection of matter of fixed identity, system can exchange heat, work with its surrounding but not exchange matter

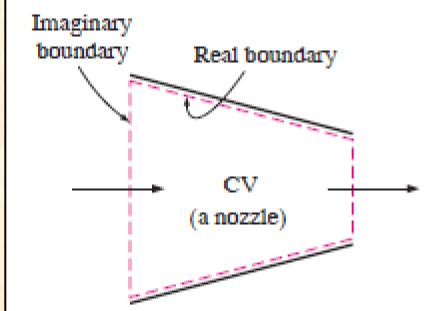
Control Volume (C.V.) (3)

Control volume is a volume in space through which fluid may flow. The boundary of the C.V. is called Control surface

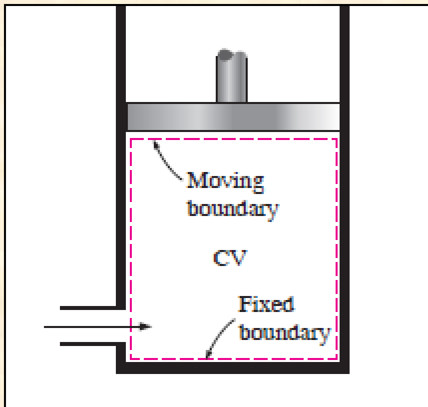
System —(Reynold Transport theorem)→ Control Volume

↑↑↑

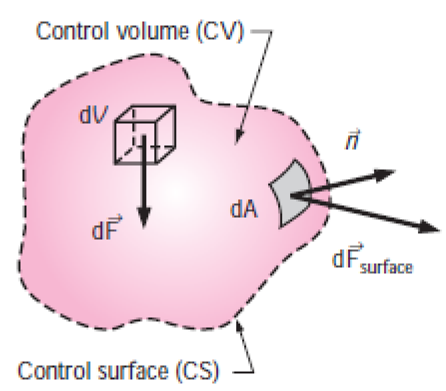
Basic laws (conservation of mass, momentum, energy)



(*a*) A control volume (CV) with real and imaginary boundaries



(*b*) A control volume (CV) with fixed and moving boundaries



3.3 Reynolds transport theorem (RTT)

Reynolds transport theorem (RTT):- states that the rate of change of an extensive property N, for the system is equal to the time rate of change of N within the control volume and the net rate of flux of the property N through the control surface.

let B represent any fluid parameters

B:
- mass $B = m$
- momentum $B = m\vec{V}$
- energy $B = me$

in general property (B) can be written as

$$B = mb \quad \text{------} (3\text{-}1)$$

where

b:
- 1 (mass)
- $\vec{V}$ (momentum)
- e (energy)

$$e = u + gz + \frac{V^2}{2}$$

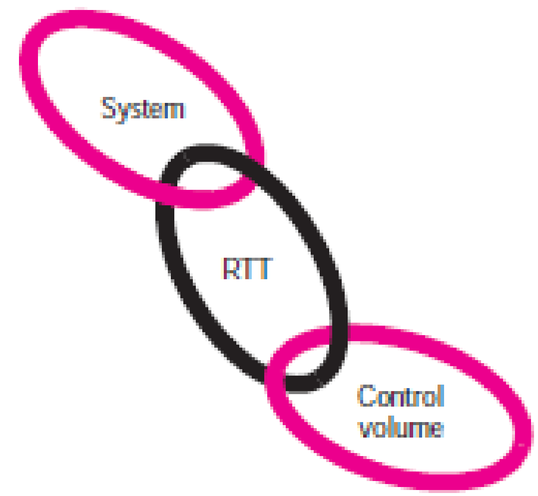
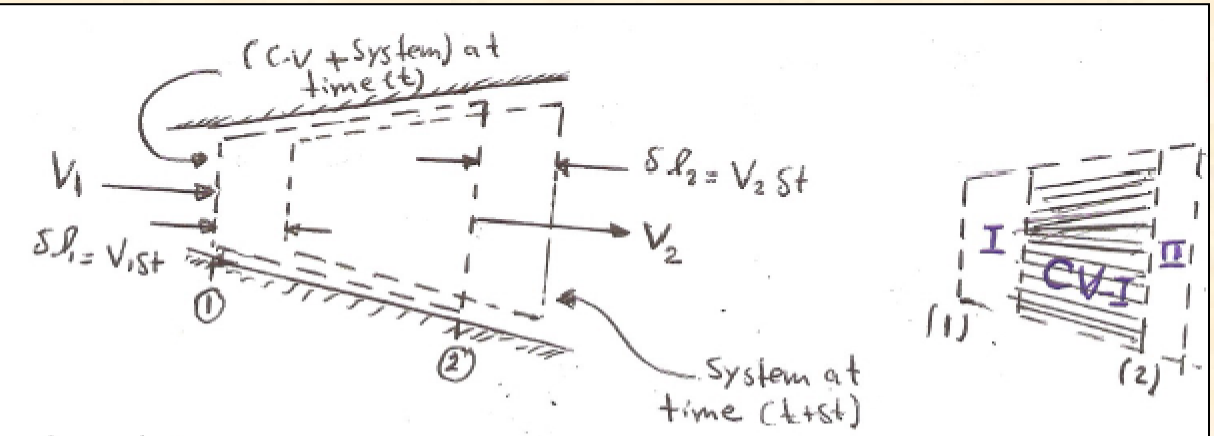
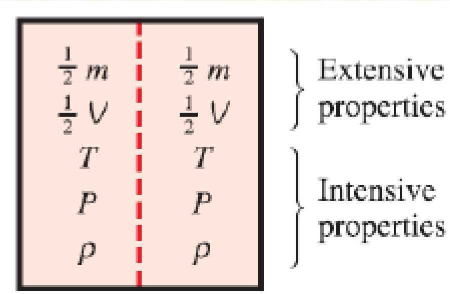


FIGURE 4–53
The Reynolds transport theorem (RTT) provides a link between the system approach and the control volume approach.





3.3 Reynolds transport theorem (RTT)

$$B_{sys}(t) = B_{c.v}(t) \quad \text{---} \quad (3\text{-}2)$$

$$B_{sys}(t+\delta t) = B_{cv}(t+\delta t) - B_{I}(t+\delta t) + B_{II}(t+\delta t) \quad \text{---} \quad (3\text{-}3)$$

Rate of change of (B) for system

$$\frac{DB_{sys}}{Dt} = \lim_{\delta t \to 0} \frac{B_{sys}(t+\Delta t) - B_{sys}(t)}{\delta t} \quad \text{---} \quad (3\text{-}4)$$

Sub equ (3-3) and (3-2) into equ (3-4) gives

$$\frac{DB_{sys}}{Dt} = \lim_{\delta t \to 0} \frac{B_{cv}(t+\delta t) - B_{I}(t+\delta t) + B_{II}(t+\delta t) - B_{cv}(t)}{\delta t}$$

$$\frac{DB_{sys}}{Dt} = \underbrace{\lim_{\delta t \to 0} \frac{B_{cv}(t+\delta t) - B_{cv}(t)}{\delta t}}_{\text{Rate of change of B within C.V}} + \lim_{\delta t \to 0} \frac{B_{II}(t+\delta t)}{\delta t} - \lim_{\delta t \to 0} \frac{B_{I}(t+\delta t)}{\delta t}$$

$$\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \lim_{\delta t \to 0} \frac{B_{II}(t+\delta t)}{\delta t} - \lim_{\delta t \to 0} \frac{B_{I}(t+\delta t)}{\delta t} \quad \text{---} \quad 3\text{-}5$$

$$B_{I}(t+\delta t) = m_1 b_1 = \underbrace{\rho_1 \delta V_1}_{m_1} b_1 = \rho_1 \underbrace{A_1 \delta \ell_1}_{\delta V_1} b_1 = \rho_1 A_1 V_1 b_1 \delta t \quad \text{---} \quad (3\text{-}6)$$

$$B_{II}(t+\delta t) = m_2 b_2 = \underbrace{\rho_2 \delta V_2}_{m_2} b_2 = \rho_2 \underbrace{A_2 \delta \ell_2}_{\delta V_2} b_2 = \rho_2 A_2 V_2 b_2 \delta t \quad \text{---} \quad (3\text{-}7)$$

Sub equ (3-6) and (3-7) into equ (3-5) gives:-

(5)

$$\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \lim_{\delta t \to 0} \frac{\rho_2 V_2 A_2 b_2 \delta t}{\delta t} - \lim_{\delta t \to 0} \frac{\rho_1 V_1 A_1 b_1 \delta t}{\delta t}$$

$$\boxed{\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \rho_2 V_2 A_2 b_2 - \rho_1 V_1 A_1 b_1} \quad \text{---} \quad (3\text{-}8)$$

3.4 Continuity equation

A continuity equation in physics is an equation that describes the transport of some quantity. It is particularly simple and powerful when applied to a conserved quantity, but it can be generalized to apply to any extensive quantity.

Here $b = 1$

Equation (3-8) becomes

$$\frac{DM_{sys}}{Dt} = \frac{\partial m_{cv}}{\partial t} + \rho_2 V_2 A_2 - \rho_1 V_1 A_1$$

by definition

$$0 = \frac{\partial m_{cv}}{\partial t} + \rho_2 V_2 A_2 - \rho_1 V_1 A_1$$

For steady flow $\frac{\partial m_{cv}}{\partial t} = 0$

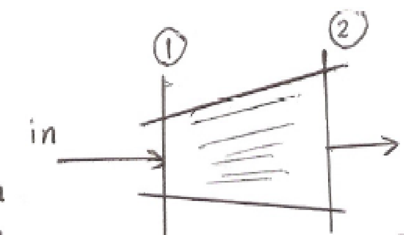
$$\boxed{\rho_2 V_2 A_2 = \rho_1 V_1 A_1} \quad \text{——— (3-9)}$$

$\dot m_2$ $\dot m_1$ = constant mass flow rate

Incompressible flow

$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 \longrightarrow \boxed{V_1 A_1 = V_2 A_2 = \text{const}}$ ——— (3-10)

$\dot V$, Q, discharge, Volume flowrate



$Q_1 = Q_2$

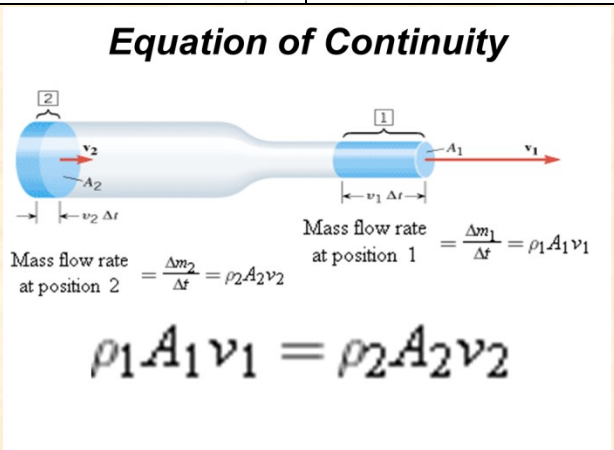
$Q = AV$

$\therefore A_1 V_1 = A_2 V_2$

For one inlet & one outlet

A: مساحة مقطع الجريان

V: سرعة الجريان



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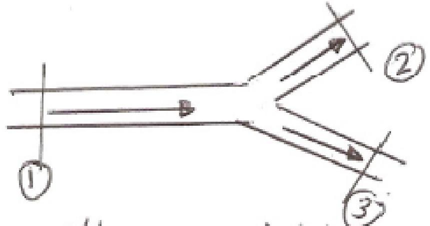
3.4 Continuity equation



For more than one inlet & one outlet

$$\sum Q_{in} = \sum Q_{out}$$

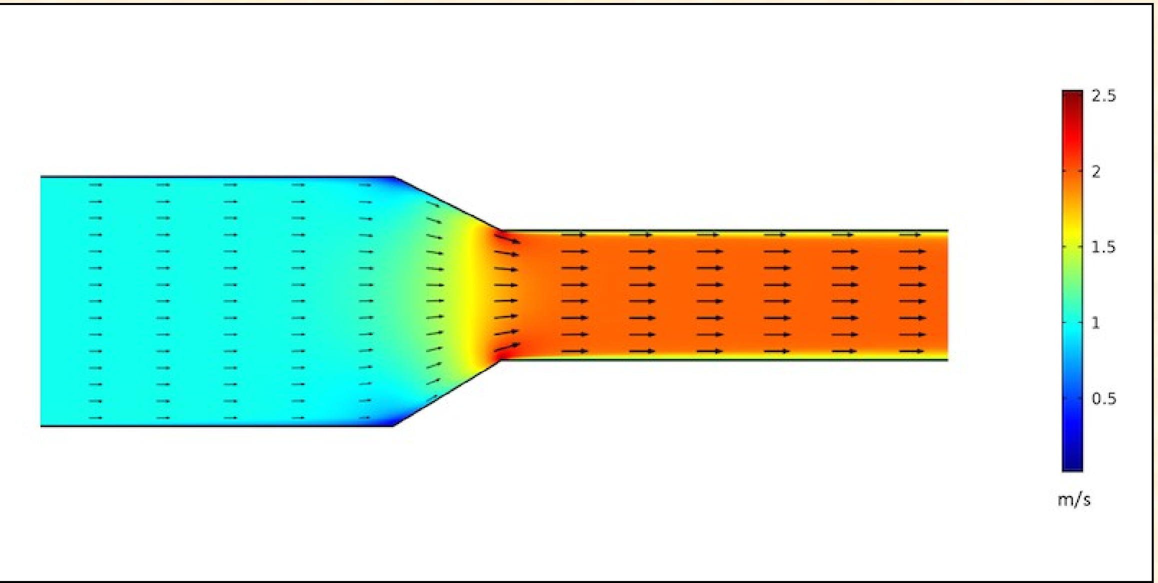
$$Q_1 = Q_2 + Q_3$$

$$A_1 V_1 = A_2 V_2 + A_3 V_3$$

Mass flow rate $= \dot{m} = \rho Q$

$$\dot{m} = \rho A v$$

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$

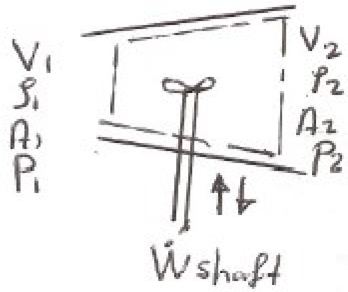


https://www.energyvanguard.com/blog/what-happens-air-flow-ducts-when-size-changes

Here $b = e$ by equation (3-8)

$$\frac{D(me)}{Dt} = \left.\frac{\partial (me)}{\partial t}\right|_{cv} + \rho_2 V_2 A_2 e_2 - \rho_1 V_1 A_1 e_1$$

steady



$$\dot{Q} - \dot{W} = \rho_2 V_2 A_2 e_2 - \rho_1 V_1 A_1 e_1$$

$$\dot{Q} - \dot{W} = \dot{m}(e_2 - e_1)$$

$$\dot{W} = P_2 A_2 V_2 - P_1 A_1 V_1 + \dot{W}_{shaft}$$

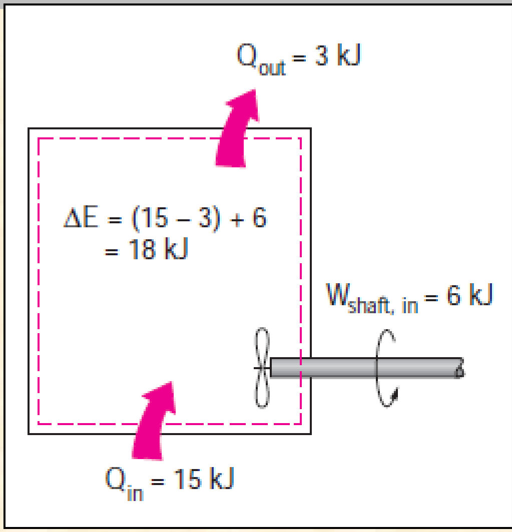
$$\therefore \dot{Q} - P_2 V_2 A_2 + P_1 V_1 A_1 - \dot{W}_{shaft} = \dot{m}\left(\left(u_2 + \frac{V_2^2}{2} + g z_2\right) - \left(u_1 + \frac{V_1^2}{2} + g z_1\right)\right) \quad \div \dot{m} g$$

$$-\frac{\dot{W}_{shaft}}{\dot{m} g} = \left(\frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2\right) - \left(\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1\right) + \left(\frac{u_2 - u_1}{g} - \frac{\dot{Q}}{\dot{m} g}\right)$$

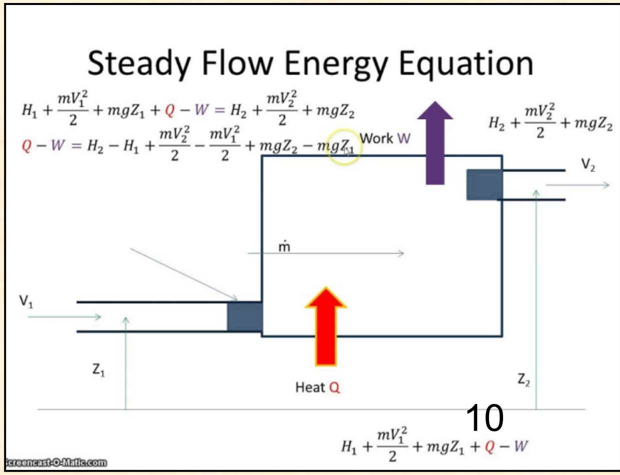
losses (frictional (h_L)) (7)

$\therefore$ incompressible flow $\rho_1 = \rho_2 = \rho$

$$\therefore \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \frac{\dot{W}_{shaft}}{\dot{m} g} + h_L \qquad (3.12)$$



The energy change of a system during a process is equal to the *net* work and heat transfer between the system and its surroundings.



3.5 Energy equation

The shaft work term is due to either a Pump or turbine

Pump

$\dot{W}_{shaft} = -\dot{W}_P$

$H_P = \frac{\dot{W}_P}{\dot{m} g}$

Turbine

$\dot{W}_{shaft} = \dot{W}_T$

$H_T = \frac{\dot{W}_T}{\dot{m} g}$

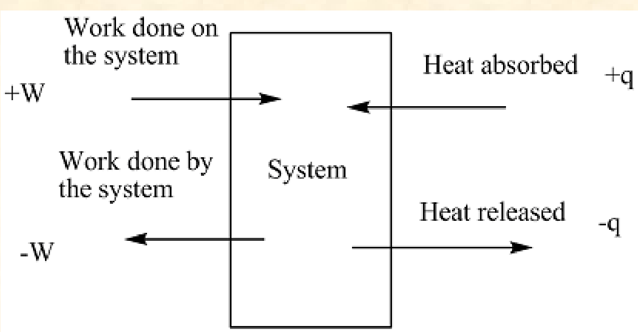
$$\dot{W}_P = h_P g \dot{m} = h_P g \rho \dot{Q} = P_P \dot{Q}$$

$$\dot{W}_T = h_T g \dot{m} = h_T g \rho \dot{Q} = P_T \dot{Q}$$

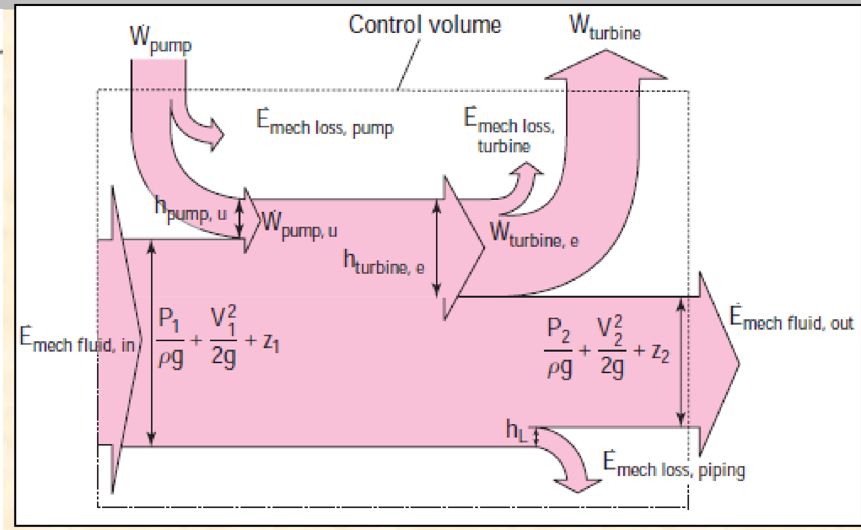
without Turbine & Pump

equ 3-12 reduce

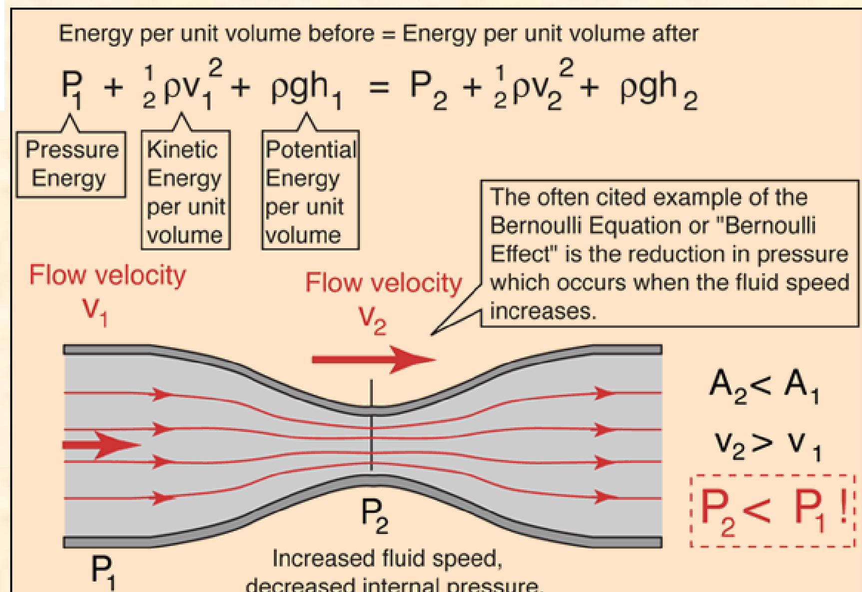
$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L \qquad \text{——— } 3\text{-}13$$



http://hyperphysics.phy-astr.gsu.edu/hbase/pber.html

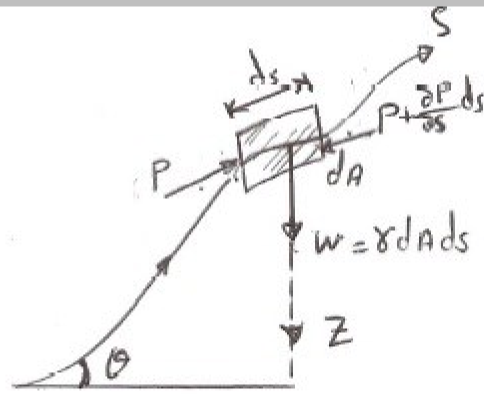


Mechanical energy flow chart for a fluid flow system that involves a pump and a turbine.



3.6 Bernoulli equation

along the streamlines direction S,
The net force acting on the element is :-



$\Sigma F_{(s)} = M a_s$ (streamline)

$$-\frac{\partial P}{\partial s}\, ds\, dA - \gamma\, dA\, ds \sin\theta = \rho\, dA\, ds\, a_s$$

$$\frac{\partial P}{\partial s} + \gamma \sin\theta + \rho a_s = 0 \quad ---- (3\text{-}14)$$

$$\sin\theta = \frac{\partial z}{\partial s}, \quad a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} + \frac{\partial V}{\partial t} = V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \quad --- (3\text{-}15)$$

from 3-14 & 3-15

$$\frac{\partial P}{\partial s} + \gamma \frac{\partial z}{\partial s} + \rho V \frac{\partial V}{\partial s} + \rho \frac{\partial V}{\partial t} = 0 \quad --- (3\text{-}16)$$

equation (3-16) Called Euler equation for unsteady flow (8)

- for steady flow $\frac{\partial}{\partial t} = 0$

∴ equ (3-16) will become

$$\frac{\partial P}{\partial s} + \gamma \frac{\partial z}{\partial s} + \rho V \frac{\partial V}{\partial s} = 0$$

- The Euler equation approximation is appropriate in high Reynolds number regions of the flow, where net viscous forces are negligible, far away from walls and wakes.
- Euler's Equation is valid for inviscid flow

Inviscid Flow

Euler's Equations of Motion

- For an inviscid flow in which all the shearing stresses are zero and the Euler's equation of motion is written as

$$\rho g_x - \frac{\partial p}{\partial x} = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

$$\rho g_y - \frac{\partial p}{\partial y} = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)$$

$$\rho g_z - \frac{\partial p}{\partial z} = \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)$$

- In vector notation Euler's equations can be expressed as

$$\rho \mathbf{g} - \nabla p = \rho\left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V}\right]$$

55

https://www.slideshare.net/ADDISUDAGNEZEGEYE/fluid-mechanics-chapter-4-differential-relations-for-a-fluid-flow

3.6 Bernoulli equation

Now S is the only independent variable, and total differentials may replace the partials

$$\frac{dP}{ds} + \gamma \frac{dz}{ds} + \rho V \frac{dV}{ds} = 0 \quad \text{——— (3-17)}$$

- homogenous flow
- multiply both side by ds
- dividing both side by γ

we get

$$\frac{dp}{\gamma} + \frac{V\,dv}{g} + dz = 0 \quad \text{——— (3-18)}$$

for incompressible Fluid $\Rightarrow \gamma = \text{const}$

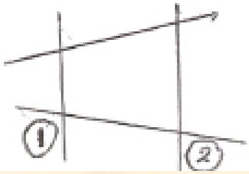
$$\int \frac{dP}{\gamma} + \int V \frac{dv}{g} + \int dz = 0$$

$$\boxed{\frac{P}{\gamma} + \frac{V^2}{2g} + Z = C} \quad \text{——— (3-19)}$$

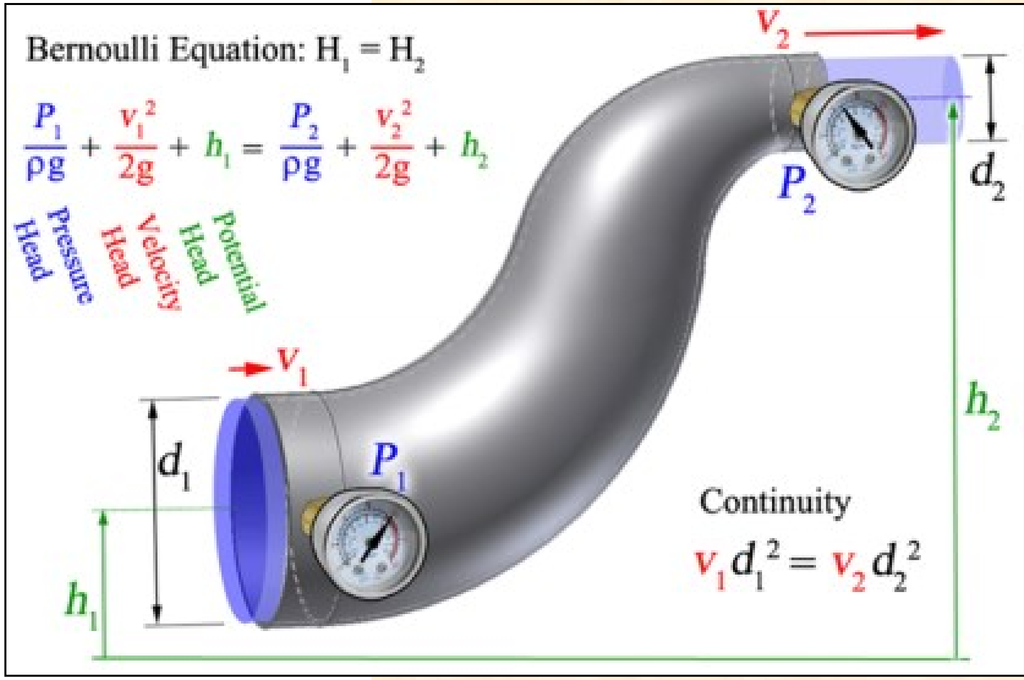
in General $\Rightarrow$ Bernoulli equation

$$\therefore \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 \quad \text{——— 3.20}$$

$\frac{P}{\gamma}$: pressure head (m)



- Bernoulli Equation is valid for steady flow



https://arvengtraining.com/wp-content/uploads/2016/11/PDI-STUDY-NOTES-TRIAL.pdf

3.6 Bernoulli equation

$\frac{V^2}{2g}$: Velocity head (m)

z : elevation (m)

with losses frictional equ 3-20 will become

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L \qquad 3\text{-}21$$

(h_L ⇓ losse frictional)

equ 3-21 is The same equ 3-13

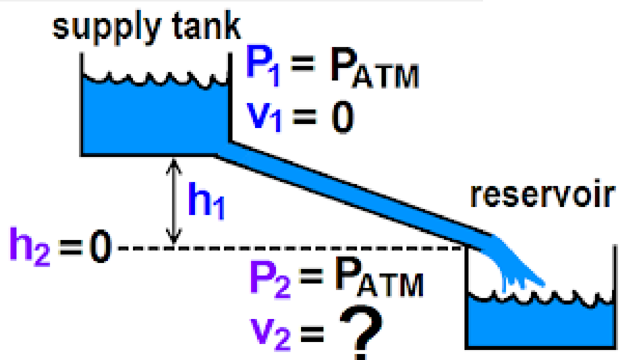
it called energy equation

معادلة الطاقة (معادلة برنولي مع الخسائر المحسوبة)

Natural Flow with Friction: Equation 1

$P_1 = P_2 = P_{ATM}$

we can cancel P_1 and P_2

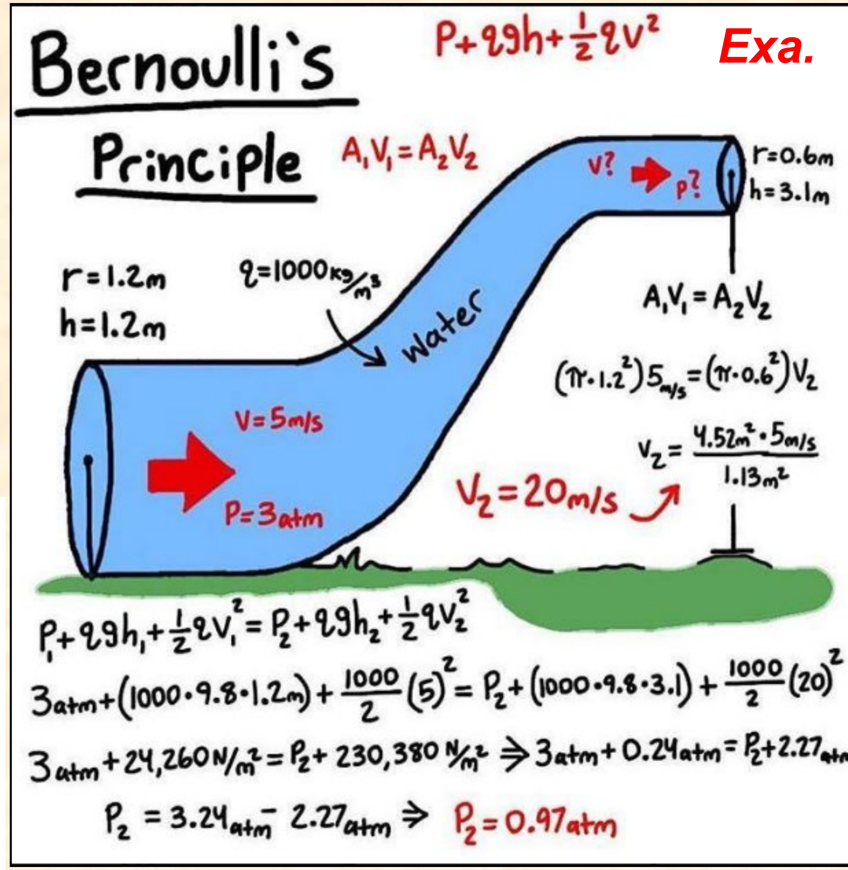


$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 + \rho g f_h$$

$\vdots$

$$v_2 = \sqrt{2g(h_1 - f_H)}$$

https://www.facebook.com/engineeringinsider/photos/a.10151263889171089/10155127631251089/?type=1&theater



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Example 1

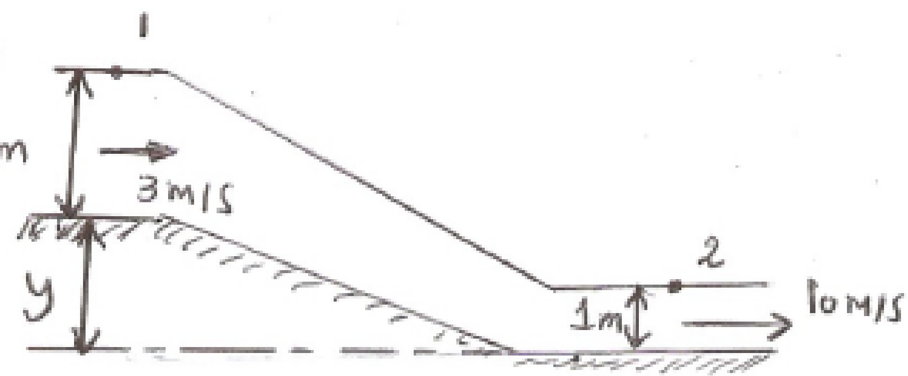
Water is flowing in an open channel at a depth of 2m and a velocity 3 m/s. It then flows down a chute into another channel where the depth is 1m and the velocity is 10m/s. Assuming frictionless flow, determine the difference in elevation of the channel floors.

Sol:-

Bernoulli's equ between ① & ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

(P_1/γ: atm, P_2/γ: atm, h_L: friction-less)



$Z_1 = y + 2$, $Z_2 = 1$, $V_1 = 3\,m/s$, $V_2 = 10\,m/s$

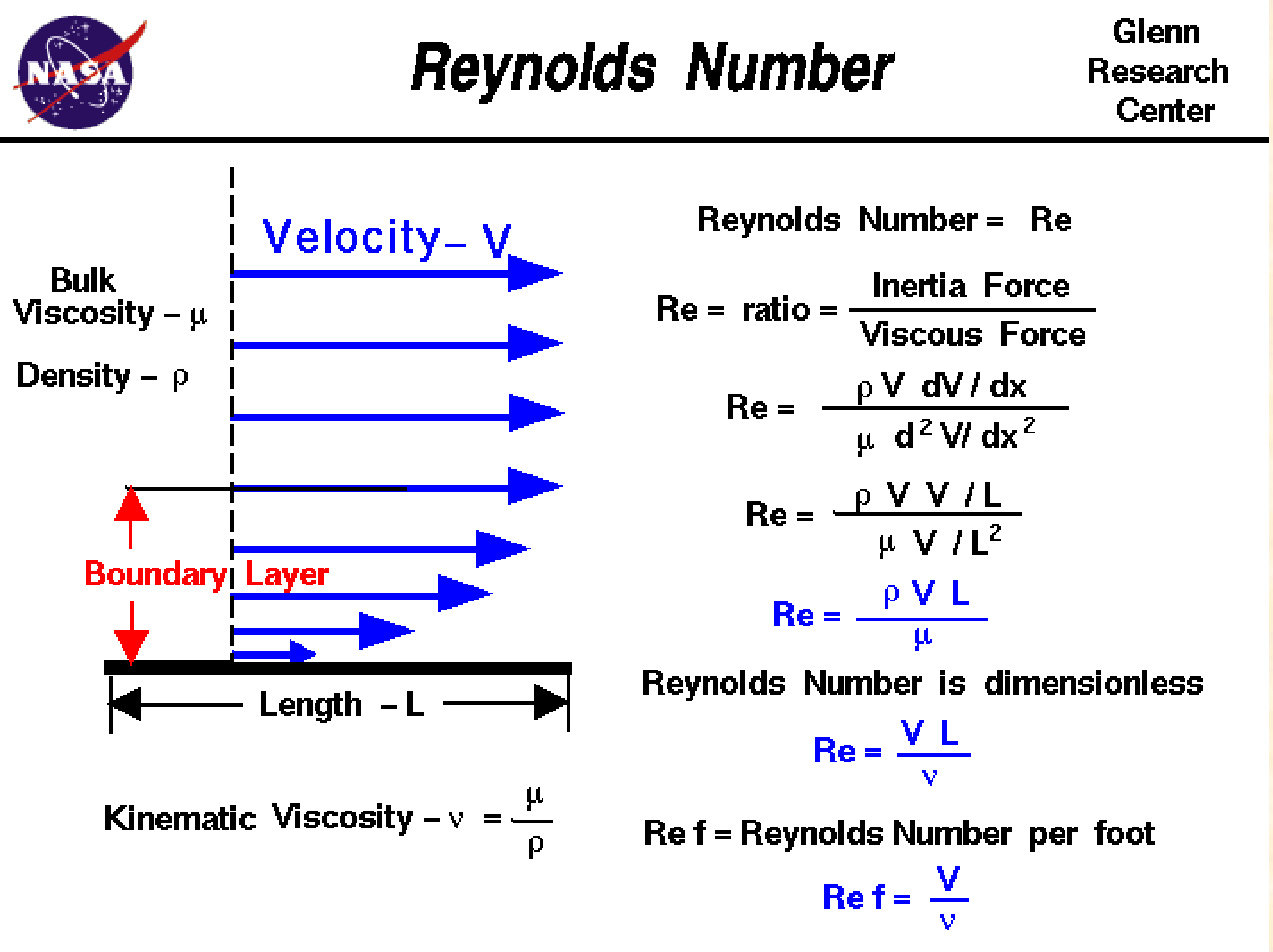
$$\therefore \frac{3^2}{2*9.81} + 0 + y + 2 = \frac{10^2}{2*9.81} + 0 + 1 \Rightarrow$$

$\boxed{y = 3.64\,m}$ ans

Examples

Example$_2$: what the Reynolds number definition and formula?

The **Reynolds number** is defined as the product of density times velocity times length divided by the viscosity coefficient. This is proportional to the ratio of inertial forces and viscous forces (forces resistant to change and heavy and gluey forces) in a fluid flow.



https://www.grc.nasa.gov/WWW/BGH/reynolds.html

Examples

Example$_3$: Air enters a 7-m-long section of a rectangular duct of cross section 15 cm × 20 cm made of commercial steel at 1atm and 35°C at an average velocity of 7 m/s. Disregarding the entrance effects, determine the fan power needed to overcome the pressure losses in this section of the duct. *Answer:* 4.9 W

Solution Air enters a rectangular duct. The fan power needed to overcome the pressure losses is to be determined.

Assumptions 1 The flow is steady and incompressible. 2 The entrance effects are negligible, and thus the flow is fully developed. 3 Air is an ideal gas. 4 The duct involves no components such as bends, valves, and connectors. 5 The flow section involves no work devices such as fans or turbines

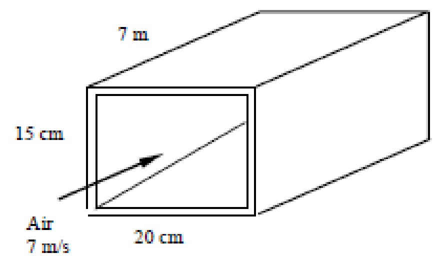
Properties The properties of air at 1 atm and 35°C are $\rho = 1.145$ kg/m^3, $\mu = 1.895\times10^{-5}$ kg/m·s, and $\nu = 1.655\times10^{-5}$ m^2/s. The roughness of commercial steel surfaces is $\varepsilon = 0.000045$ m.

Analysis The hydraulic diameter, the volume flow rate, and the Reynolds number in this case are

$$D_h = \frac{4A_c}{p} = \frac{4ab}{2(a+b)} = \frac{4(0.15\text{ m})(0.20\text{ m})}{2(0.15+0.20)\text{ m}} = 0.1714\text{ m}$$

$$\dot{V} = VA_c = V(a\times b) = (7\text{ m/s})(0.15\times0.20\text{ m}^2) = 0.21\text{ m}^3/\text{s}$$

$$\text{Re} = \frac{\rho VD_h}{\mu} = \frac{(1.145\text{ kg/m}^3)(7\text{ m/s})(0.1714\text{ m})}{1.895\times10^{-5}\text{ kg/m}\cdot\text{s}} = 72{,}490$$



which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon / D_h = \frac{4.5\times10^{-5}\text{ m}}{0.1714\text{ m}} = 2.625\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon / D_h}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{2.625\times10^{-4}}{3.7} + \frac{2.51}{72{,}490\sqrt{f}}\right)$$

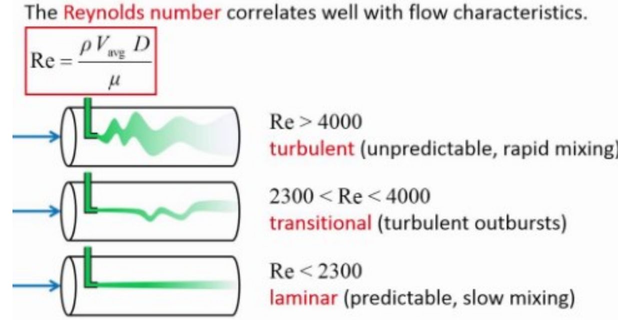
It gives $f = 0.02034$. Then the pressure drop in the duct and the required pumping power become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.02034\frac{7\text{ m}}{0.1714\text{ m}}\frac{(1.145\text{ kg/m}^3)(7\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ Pa}}{1\text{ N/m}^2}\right) = 23.3\text{ Pa}$$

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = \left(0.21\text{ m}^3/\text{s}\right)\left(23.3\text{ Pa}\right)\left(\frac{1\text{ W}}{1\text{ Pa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{4.90\ W}$$

Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give $f = 0.02005$, which is sufficiently close to 0.02034. Also, the power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be much more than this due to fan inefficiency; the electrical power input will be even more due to motor inefficiency.

The Reynolds number correlates well with flow characteristics.



Examples

Example$_4$: What is hydraulic diameter? How is it defined? What is it equal to for a circular pipe of diameter D?

Solution We are to define and discuss hydraulic diameter.

Analysis For flow through non-circular tubes, the Reynolds number and the friction factor are based on the *hydraulic diameter* D_h defined as $D_h = \frac{4A_c}{p}$ where A_c is the cross-sectional area of the tube and p is its perimeter. The hydraulic diameter is defined such that it reduces to ordinary diameter D for circular tubes since $D_h = \frac{4A_c}{p} = \frac{4\pi D^2/4}{\pi D} = D$.

Discussion Hydraulic diameter is a useful tool for dealing with non-circular pipes (e.g., air conditioning and heating ducts in buildings).

Example$_5$: True or false: For each statement, choose whether the statement is true or false and discuss your answer briefly.

(a) The Reynolds transport theorem is useful for transforming conservation equations from their naturally occurring control volume forms to their system forms.

False: The statement is backwards, since the conservation laws are naturally occurring in the system form.

(*b*) The Reynolds transport theorem is applicable only to nondeforming control volumes.

False: The RTT can be applied to any control volume, fixed, moving, or deforming.

(*c*) The Reynolds transport theorem can be applied to both steady and unsteady flow fields.

True: The RTT has an unsteady term and can be applied to unsteady problems.

(*d*) The Reynolds transport theorem can be applied to both scalar and vector quantities.

True: The extensive property *B* (or its intensive form *b*) in the RTT can be any property of the fluid – scalar, vector, or even tensor.

Examples

Example$_6$: Explain the importance of the Reynolds transport theorem in fluid mechanics, and describe how the linear momentum equation is obtained from it.

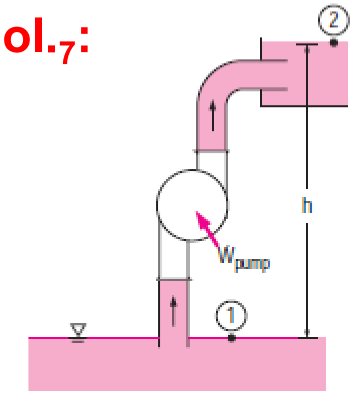
Solution We are to discuss the importance of the RTT, and its relationship to the linear momentum equation.

Analysis **The relationship between the time rates of change of an extensive property for a system and for a control volume is expressed by the** *Reynolds transport theorem* (RTT), which provides the link between the system and control volume concepts. The linear momentum equation is obtained by setting $b = \vec{V}$ **and thus** $B = m\vec{V}$ **in the Reynolds transport theorem.**

Discussion Newton's second law applies directly to a system of fixed mass, but we use the RTT to transform from the system formulation to the control volume formulation.

Example$_7$: in mechmical system in Fig (5-20), calculate the pump work that using for pushing the water to the tank at point 2? For more explaining Most fluid flow problems involve mechanical forms of energy only, and such problems are conveniently solved by using a *mechanical energy* balance.

Sol.$_7$:



Steady flow

$V_1 = V_2$

$z_2 = z_1 + h$

$P_1 = P_2 = P_{atm}$

$$\dot{E}_{mech,\ in} = \dot{E}_{mech,\ out} + \dot{E}_{mech,\ loss}$$

$$\dot{W}_{pump} + \dot{m}gz_1 = \dot{m}gz_2 + \dot{E}_{mech,\ loss}$$

$$\dot{W}_{pump} = \dot{m}gh + \dot{E}_{mech,\ loss}$$

FIGURE 5–20
Most fluid flow problems involve mechanical forms of energy only, and such problems are conveniently solved by using a mechanical energy balance.

Examples

Example$_8$: Express the Bernoulli equation in three different ways using (*a*) energies, (*b*) pressures, and (*c*) heads.

Solution We are to express the Bernoulli equation in three different ways.

Analysis The Bernoulli equation is expressed in three different ways as follows:

(*a*) In terms of energies:

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

(*b*) In terms of pressures:

$$P + \rho\frac{V^2}{2} + \rho g z = \text{constant}$$

(*c*) in terms of heads:

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = H = \text{constant}$$

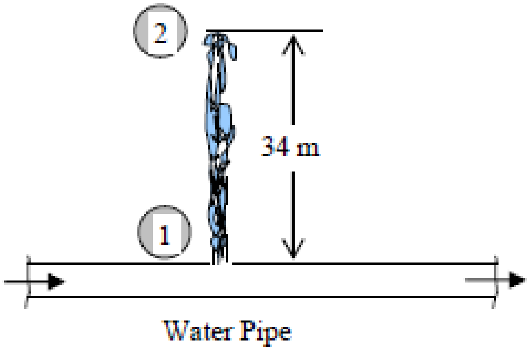
Examples

Example$_9$: In cold climates, water pipes may freeze and burst if proper precautions are not taken. In such an occurrence, the exposed part of a pipe on the ground ruptures, and water shoots up to 34 m. Estimate the gage pressure of water in the pipe. State your assumptions and discuss if the actual pressure is more or less than the value you predicted.

Solution A water pipe bursts as a result of freezing, and water shoots up into the air a certain height. The gage pressure of water in the pipe is to be determined.

Assumptions **1** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** The water pressure in the pipe at the burst section is equal to the water main pressure. **3** Friction between the water and air is negligible. **4** The irreversibilities that may occur at the burst section of the pipe due to abrupt expansion are negligible.

Properties We take the density of water to be 1000 kg/m^3.



Analysis This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. The water height will be maximum under the stated assumptions. The velocity inside the hose is relatively low ($V_1 \cong 0$) and we take the burst section of the pipe as the reference level ($z_1 = 0$). At the top of the water trajectory $V_2 = 0$, and atmospheric pressure pertains. Then the Bernoulli equation simplifies to

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_1}{\rho g}=\frac{P_{atm}}{\rho g}+z_2 \quad\rightarrow\quad \frac{P_1-P_{atm}}{\rho g}=z_2 \quad\rightarrow\quad \frac{P_{1,gage}}{\rho g}=z_2$$

Solving for $P_{1,gage}$ and substituting,

$$P_{1,gage}=\rho g z_2=(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(34\ \text{m})\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)=334\ \text{kPa}$$

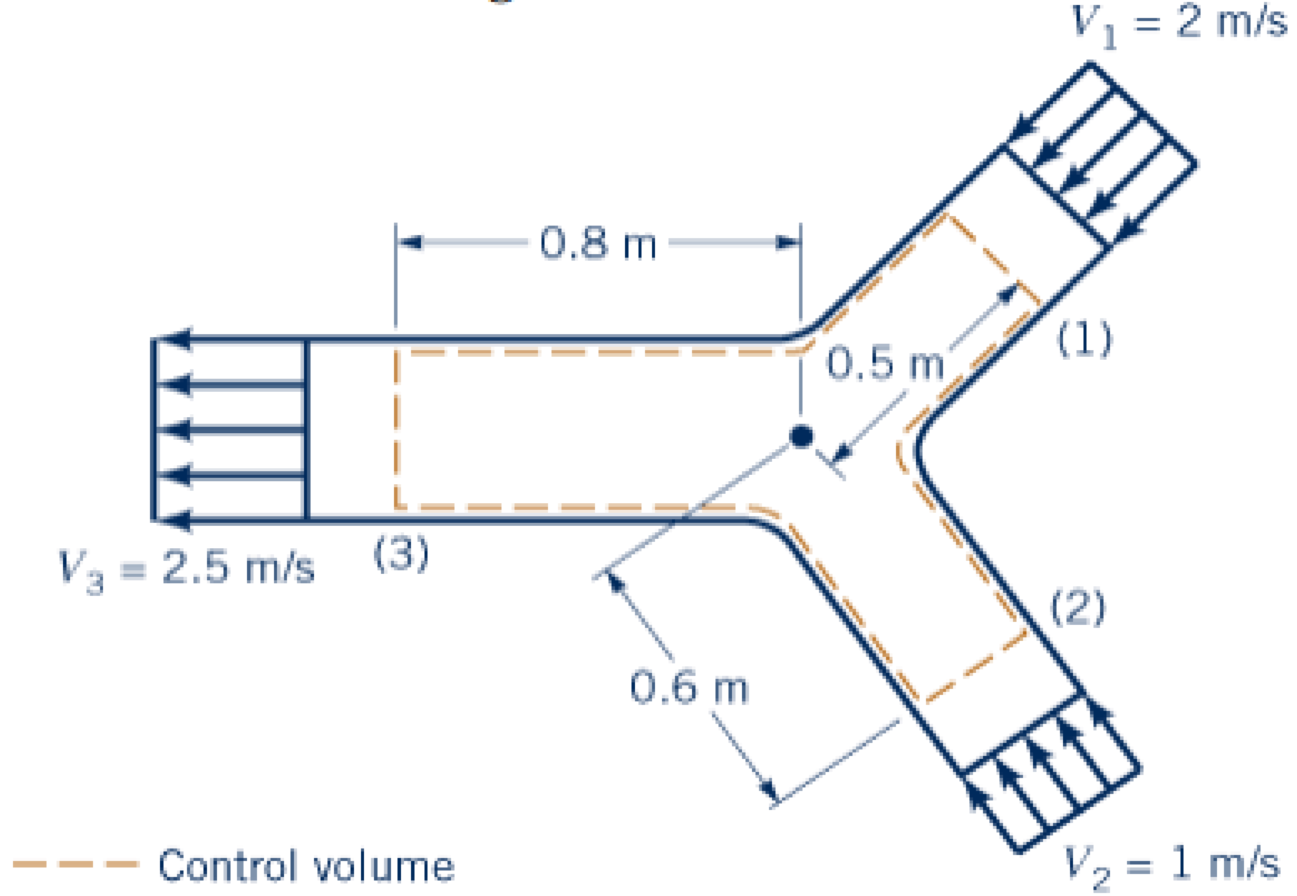
Therefore, the pressure in the main must be at least 334 kPa above the atmospheric pressure.

Discussion The result obtained by the Bernoulli equation represents a limit, since frictional losses are neglected, and should be interpreted accordingly. It tells us that the water pressure (gage) cannot possibly be less than 334 kPa (giving us a lower limit), and in all likelihood, the pressure will be much higher.

Homeworks (1):

Water flows in the branching pipe with uniform velocity at each inlet and outlet. The fixed control volume indicated coincides with the system at time t_0. Make a sketch to indicate (a) the boundary of the system at time $t=t_0+0.2$ s; (b) the fluid that left the control volume during that 0.2-s interval, and (c) the fluid that entered the control volume during that time interval.

HW1:



Homeworks (1):

Hw$_2$: The Force to Hold a Deflector Elbow in Place

A reducing elbow is used to deflect water flow at a rate of 14 kg/s in a horizontal pipe upward 30° while accelerating it (Fig. 6–20). The elbow discharges water into the atmosphere. The cross-sectional area of the elbow is 113 cm2 at the inlet and 7 cm2 at the outlet. The elevation difference between the centers of the outlet and the inlet is 30 cm. The weight of the elbow and the water in it is considered to be negligible. Determine (*a*) the gage pressure at the center of the inlet of the elbow and (*b*) the anchoring force needed to hold the elbow in place.

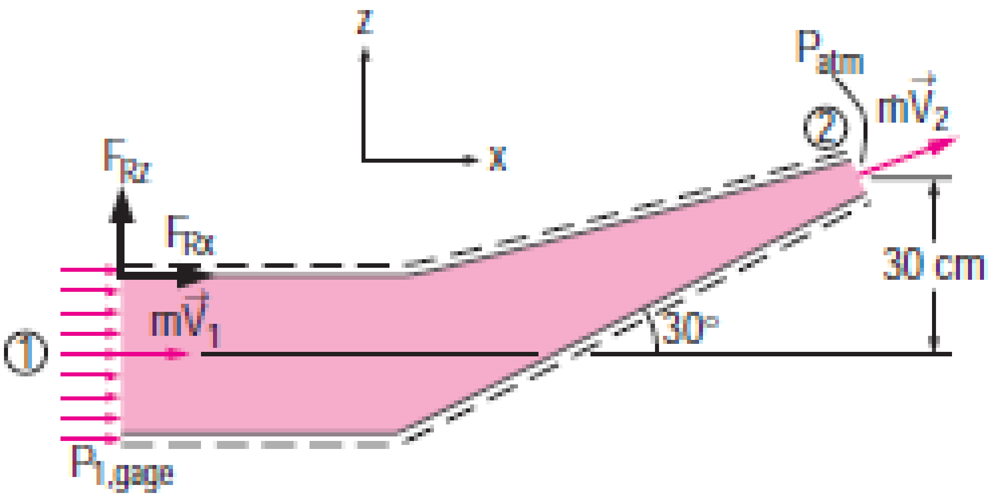


FIGURE 6–20
Schematic for Example 6–2.

Homeworks (1):

Hw$_3$: Pumping Power and Frictional Heating in a Pump

The pump of a water distribution system is powered by a 15-kW electric motor whose efficiency is 90 percent (Fig. 5–54). The water flow rate through the pump is 50 L/s. The diameters of the inlet and outlet pipes are the same, and the elevation difference across the pump is negligible. If the pressures at the inlet and outlet of the pump are measured to be 100 kPa and 300 kPa (absolute), respectively, determine (*a*) the mechanical efficiency of the pump and (*b*) the temperature rise of water as it flows through the pump due to the mechanical inefficiency.

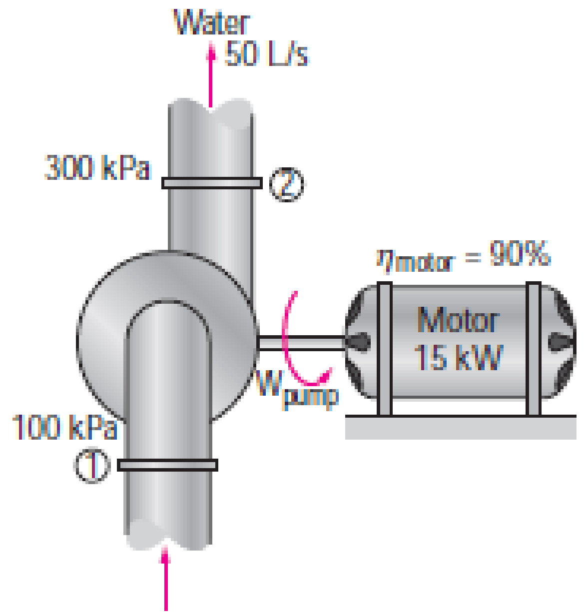


FIGURE 5–54
Schematic for Example 5–12.

Homeworks (2):

Hw$_4$: Fan Selection for Air Cooling of a Computer

A fan is to be selected to cool a computer case whose dimensions are 12 cm * 40 cm * 40 cm (Fig. 5–56). Half of the volume in the case is expected to be filled with components and the other half to be air space. A 5-cmdiameter hole is available at the back of the case for the installation of the fan that is to replace the air in the void spaces of the case once every second. Small low-power fan–motor combined units are available in the market and their efficiency is estimated to be 30 percent. Determine (*a*) the wattage of the fan–motor unit to be purchased and (*b*) the pressure difference across the fan. Take the air density to be 1.20 kg/m^3.

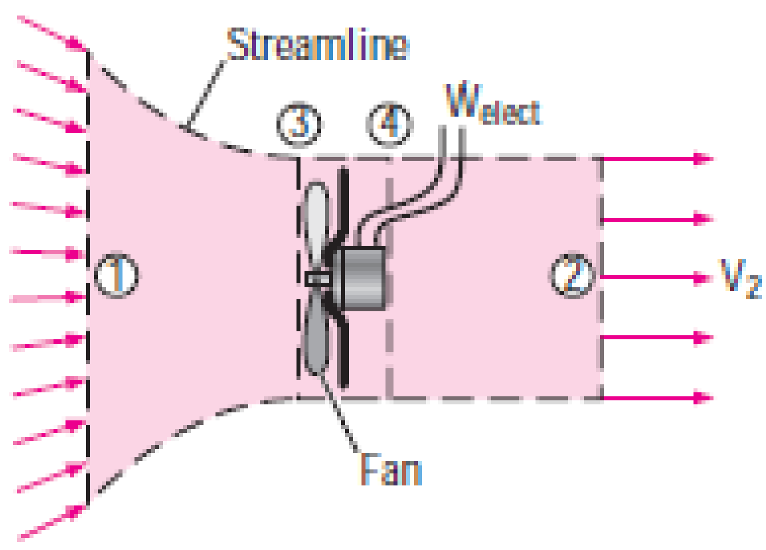
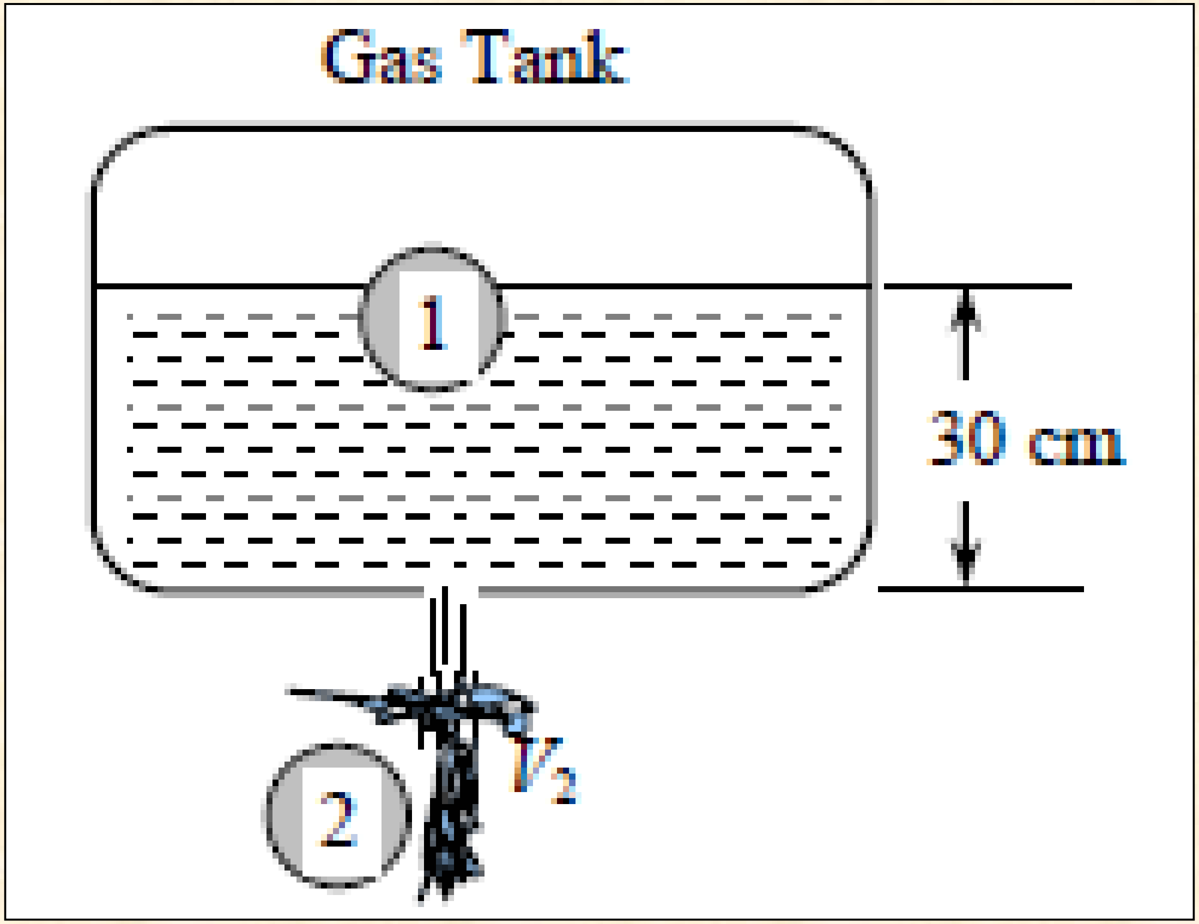


FIGURE 5–56
Schematic for Example 5–14.

Homeworks (2):

Hw$_5$: While traveling on a dirt road, the bottom of a car hits a sharp rock and a small hole develops at the bottom of its gas tank. If the height of the gasoline in the tank is 30 cm, determine the initial velocity of the gasoline at the hole. Discuss how the velocity will change with time and how the flow will be affected if the lid of the tank is closed tightly. *Answer:* 2.43 m/s



Exams and Grading Policy:

❖ The distribution of Fluid mechanics degree for the students in course-1 as following the table:

Quizzes (2) 10%		**Project (1)** 6%		**Assignments (H.Ws)** 9%		**Midterm exam (1)** 10%	**Laboratory (3)** 15%		**Final**
(1)	(2)	Report Structures	Report Discussion	Online (5)	Onsite (2)	-	Report Structures	Report Discussion	
5 %	**5 %**	4 %	2 %	**5 %**	4 %	10 %	9 %	6 %	**50 %**

- Note: Solve all five Homeworks and sending me the answering next week on Thursday (Hw1: 6 February 2025) & (Hw2: 13 February 2025).
- I hope everything is clear for all students

Good luck